

# On the Effect of Echoes in Hybrid Terrestrial-Satellite Single Frequency Networks: Analysis and Countermeasures

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**Abstract**—Hybrid terrestrial-satellite Single Frequency Networks (SFN) achieve large spectral efficiencies due to a higher frequency reuse, which is attained by transmitting the same waveform in the same frequency band from satellite and terrestrial transmitters. However, the presence of multiple transmitters propitiates the existence of the so-called *SFN echoes*, which can degrade the system performance even if they arrive within the Orthogonal Frequency Division Multiplexing (OFDM) guard interval. In this paper we characterize this effect by resorting to Packet Error Rate (PER) prediction metrics (or *effective Signal to Noise Ratio (SNR)* metrics), and analyze two simple preprocessing schemes that mitigate this degradation: the use of Alamouti space-time codes, and a convenient pre-filtering at the terrestrial transmitter.

**Index Terms**—Single Frequency Network; Hybrid Terrestrial-Satellite; Effective SNR; Space-Time Coding; Satellite OFDM

## I. INTRODUCTION

During the last few years, the increasing demand of mobile multimedia services has forced the telecommunication operators to deploy sophisticated network structures to offer high bit rates with adequate availability. In satellite communications, the mobility of the end users is a problem of special magnitude, as it could cause Line Of Sight (LOS) *blockage*, thus severely degrading communication performance.

In order to cope with this problem, the insertion of terrestrial transmitters in the satellite network has been recently found to be of special interest: while the satellite link is used to cover large areas where direct vision is possible (e.g. rural zones), reception is reinforced by terrestrial transmitters in those places where LOS reception from the satellite is difficult or is expected to suffer frequent blockage (e.g. cities with large buildings).

However, the deployment of terrestrial transmitters has to be carefully planned in order to avoid interference with the satellite link. This issue can be easily overcome by splitting the available bandwidth into two bands, using one of them for the terrestrial network and the other one to the satellite, thus conforming a Multiple Frequency Network (MFN). This approach decreases the overall spectral efficiency of the system

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and adds additional complexity in the receivers, as they have to monitor two different bands, choosing afterwards the one that provides a higher quality (selection combining), or even to demodulate both signals to exploit all the available information (by applying maximal ratio combining, for example). Several combining strategies with different degrees of performance and complexity are proposed for the MFN operation of DVB-SH in [1].

An alternative approach to the transmission of the terrestrial and satellite contributions in different frequency bands is the use of Orthogonal Frequency Division Multiplexing (OFDM), so satellite and terrestrial transmitters can use the same frequency band in Single Frequency Network (SFN) operation. This network architecture requires time and frequency synchronization among the different transmitters, and an overall channel length (including both the usual multipath and the replicas coming from the different transmitters) smaller than that of the Cyclic Prefix (CP). This kind of operation is enabled in DVB-SH [2] and is expected to be a key feature in future satellite communication systems.

Although it is well known that the presence of SFN echoes can severely degrade the system performance provided they arrive outside the guard interval, the effect of these echoes when they arrive inside it is usually ignored, or simply assumed to result in a power gain [3]. However, it has been empirically shown that the presence of echoes is harmful for scenarios with a strong LOS reception, while those receivers experiencing a strong multipath benefit from the diversity created by the different transmitters [4]. The effect of the echoes in LOS scenarios is the creation of a *ripple* effect in the frequency domain, which we refer to as *channel degradation*. The objective of this paper is to quantify the effect of both *power gain* and *channel degradation*, analyze the performance gain (or loss) caused by the insertion of a terrestrial transmitter, and propose different countermeasures to overcome the *channel degradation*. The results of this work can be useful for the analysis and design of broadcast hybrid terrestrial satellite SFN.

We will characterize the impact of the echoes resorting to the use of Packet Error Rate prediction metrics, also known as *Effective SNR Metrics* [5], [6]. The objective of these metrics is to predict the PER of an OFDM receiver in the presence

of a general fading channel, as it is well known that the performance of a multicarrier receiver is not only a function of the average received SNR, but also of the particular channel seen by the different carriers.

The remaining of the paper is structured as follows: Section II presents the system model, and introduces the notation and metrics to be used in the upcoming analysis; in Section III a performance analysis is carried out for the usual SFN operation, i.e., when no channel degradation countermeasures are employed; Section IV and V study the use of Alamouti preprocessing and filtering, respectively; Section VI presents the results and the verification of the analytical expressions via simulation; finally, Section VII concludes the paper.

## II. SYSTEM MODEL

In this paper we will study a scenario where a receiver is exposed to the OFDM signal coming from the satellite and a single terrestrial transmitter (also known as Complementary Ground Component - CGC). We will always assume perfect synchronization and channel estimation, and an overall channel shorter than the CP. The (time domain) received baseband signal after CP removal can be written as

$$y_n = (h_n + g_n) \circledast x_n + w_n \quad (1)$$

with  $h_n$  and  $g_n$  the time response of the channels from the satellite and CGC, respectively,  $x_n$  the time domain signal, normalized to have unit power,  $w_n \sim \mathcal{CN}(0, \sigma^2)$  a circularly symmetric white gaussian noise sample, and  $\circledast$  the circular convolution operator. If we assume an OFDM system with  $N$  carriers, equation (1) can be recast in the Discrete Fourier Transform (DFT) domain as

$$Y_k = (H_k + G_k) X_k + W_k \quad (2)$$

with  $H_k$ ,  $G_k$ ,  $X_k$  and  $W_k$  the  $N$ -points DFT of  $h_n$ ,  $g_n$ ,  $x_n$  and  $w_n$ . In Figure 1 there is a plot summarizing the system model.

We define the Average SNR Metric (ASM) of the hybrid system as

$$\bar{\gamma}_H \doteq \frac{1}{N} \sum_{k=1}^N \frac{|H_k + G_k|^2}{\sigma^2} \approx \frac{1}{N} \sum_{k=1}^N \frac{|H_k|^2 + |G_k|^2}{\sigma^2} \quad (3)$$

where the approximation holds if both  $H_k$  and  $G_k$  are independently drawn from a zero mean probability distribution<sup>1</sup>, so  $E\{H_k G_k^*\} = 0$ , where  $(\cdot)^*$  denotes the complex conjugation operator.

In the same way, the ASM in absence of the CGC is

$$\bar{\gamma}_S \doteq \frac{1}{N} \sum_{k=1}^N \frac{|H_k|^2}{\sigma^2}. \quad (4)$$

If we use the ASM as a performance metric, it is clear that  $\bar{\gamma}_H \geq \bar{\gamma}_S$ , so we conclude that the SFN operation

<sup>1</sup>This is a good approximation even in the case of systems working with a strong Line of Sight, as a uniform phase term in the signal received from one of the transmitters makes the resulting ASM to follow (3).

always improves the system performance. This is the usual approximation when calculating the SFN gain [3].

However, the performance of multicarrier systems is not just a direct function of the ASM, but also of the distribution of the SNR on the different carriers. Effective SNR Metrics (ESM) have been developed [5], [6] with the purpose of predicting the performance (in terms of Packet Error Rate - PER) of a multicarrier system in the presence of a frequency selective channel. The effective SNR  $\hat{\gamma}$  can be written as a function of the SNR of the  $N$  carriers ( $\gamma_i$ ,  $i = 1, \dots, N$ ) as

$$\hat{\gamma} \doteq \Theta^{-1} \left( \frac{1}{N} \sum_{k=1}^N \Theta(\gamma_i) \right) \quad (5)$$

where the function  $\Theta$  is chosen as a concave increasing function, or convex decreasing function.

In particular, the Mutual Information ESM (MIESM) has been found of special interest because of its accuracy in predicting the PER [7]. The function  $\Theta$  associated to the MIESM<sup>2</sup>, taken from [8], is

$$\Theta(\gamma) = \frac{1}{M \log_2 M} \sum_{m=1}^M E_U \left\{ \log_2 \left( \sum_{k=1}^M e^{-\frac{|X_m - X_k + U| - |U|^2}{1/\gamma}} \right) \right\} \quad (6)$$

where  $U \sim \mathcal{CN}(0, 1/\gamma)$ , and  $X_m$ ,  $m = 1, \dots, M$  are the complex constellation points. (6) can be approximated as [7]

$$\Theta(\gamma) = \sum_{l=1}^L \phi_l e^{-\beta_l \gamma} \quad (7)$$

where  $\sum_{l=1}^L \phi_l = 1$ , and  $\phi_l \geq 0$  and  $\beta_l \geq 0$  are parameters that have to be properly chosen in order to fit the actual value of (6).

Following this ESM approach, and similarly to the definition of (3) and (4), we define the ESM of the hybrid and only-satellite scenarios as

$$\hat{\gamma}_H \doteq \Theta^{-1} \left( \frac{1}{N} \sum_{k=1}^N \Theta \left( \frac{|H_k + G_k|^2}{\sigma^2} \right) \right) \quad (8)$$

and

$$\hat{\gamma}_S \doteq \Theta^{-1} \left( \frac{1}{N} \sum_{k=1}^N \Theta \left( \frac{|H_k|^2}{\sigma^2} \right) \right). \quad (9)$$

Throughout the paper, we will use the ESM as a performance metric, and define the ESM gain as the ratio between the hybrid ESM and the only-satellite ESM:

$$\Delta \hat{\gamma} = \frac{\hat{\gamma}_H}{\hat{\gamma}_S}. \quad (10)$$

With this definition, if we have  $\Delta \hat{\gamma} > 1$ , then the system benefits from the insertion of the CGC. Conversely, if  $\Delta \hat{\gamma} < 1$ , then the *channel degradation* has a dominant effect, and hybrid performance is worse than that of the only-satellite system.

<sup>2</sup>For the sake of simplicity, (6) is different (in a constant term) from the original expression in [8], but the overall ESM is the same.

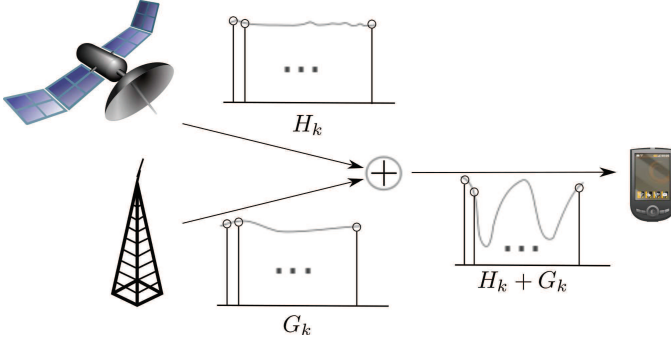


Fig. 1. System model: the channels  $H_k$  and  $G_k$ , despite being almost flat, form a frequency selective channel after its addition.

### III. NO PREPROCESSING

In this section we will obtain the ESM of the hybrid and only-satellite scenarios for the AWGN and Rician channels. The AWGN derivation is presented first for the sake of completeness, although it can be obtained as a particular case of the general Rician channel.

#### A. AWGN Channel

In the AWGN case, we can set  $H_k = 1$ ,  $k = 1, \dots, N$ , and

$$G_k = \alpha e^{-j(\theta + 2\pi n_0 \frac{k}{N})} \quad (11)$$

where  $\alpha$  accounts for the different amplitude of  $G_k$ ,  $n_0$  accounts for the delay between the two contributions, and  $\theta$  is the difference between phases. Clearly, since  $H_k = 1$ , we have

$$\hat{\gamma}_S = \bar{\gamma}_S = \frac{1}{\sigma^2}. \quad (12)$$

The calculation of  $\hat{\gamma}_H$  is more involved. First, note that

$$|H_k + G_k|^2 = 1 + \alpha^2 + 2\alpha \cos\left(\theta + 2\pi n_0 \frac{k}{N}\right), \quad (13)$$

so in the degenerate case of  $n_0 = 0, \theta = 0$  we have  $|H_k + G_k|^2 = (1 + \alpha)^2 \forall k$ , and if  $n_0 = 0, \theta = \pi$ ,  $|H_k + G_k|^2 = (1 - \alpha)^2 \forall k$ . However, for usual values of  $n_0$ , the  $N$  different arguments of the cosine

$$\theta + 2\pi n_0 \frac{k}{N}, \quad k = 1 \dots, N, \quad (14)$$

will conform an approximately uniform sampling of the interval  $(0, 2\pi]$ , so we can write for a sufficiently large number of carriers

$$\Theta(\hat{\gamma}_H) = \frac{1}{N} \sum_{k=1}^N \Theta\left(\frac{|H_k + G_k|^2}{\sigma^2}\right) \approx E_a \left\{ \Theta\left(\frac{1 + \alpha^2 + 2\alpha \cos(a)}{\sigma^2}\right) \right\} \quad (15)$$

with  $a \sim \mathcal{U}(0, 2\pi]$ . Substituting (7) in (15) we arrive to

$$\Theta(\hat{\gamma}_H) = \frac{1}{2\pi} \int_0^{2\pi} \sum_{l=1}^L \phi_l e^{-\beta_l (1 + \alpha^2 + 2\alpha \cos(a)) / \sigma^2} da = \frac{1}{2\pi} \int_0^{2\pi} \sum_{l=1}^L \phi_l e^{-\beta_l (1 + \alpha^2) / \sigma^2} I_0\left(\frac{2\beta_l \alpha}{\sigma^2}\right) \quad (16)$$

with  $I_0(\cdot)$  the zeroth order modified Bessel function of the first kind.

In order to gain insight on the implications of (16), we will focus on the case<sup>3</sup> with  $L = 1$  (or, equivalently, the Exponential ESM - EESM [5]), so

$$\Theta^{-1}(x) = -\frac{1}{\beta} \log(x) \quad (17)$$

where we denote  $\beta \doteq \beta_1$  for the sake of simplicity. Note that  $\phi_1 = 1$ , so a simple form for  $\hat{\gamma}_H$  is obtained just by applying (17) to (16)

$$\hat{\gamma}_H = -\frac{1}{\beta} \log\left(e^{-\beta(1+\alpha^2)/\sigma^2} I_0\left(\frac{2\beta\alpha}{\sigma^2}\right)\right) = \frac{1 + \alpha^2}{\sigma^2} - \frac{1}{\beta} \log\left(I_0\left(\frac{2\beta\alpha}{\sigma^2}\right)\right), \quad (18)$$

so, by just dividing (18) by (12), the ESG for the AWGN channel reads as

$$\Delta\hat{\gamma} = 1 + \alpha^2 - \frac{\sigma^2}{\beta} \log\left(I_0\left(\frac{2\beta\alpha}{\sigma^2}\right)\right). \quad (19)$$

This expression has two clearly differentiated components:

- The term  $1 + \alpha^2 = \Delta\bar{\gamma}$  represents the average power gain due to the two different components.
- The term  $\frac{\sigma^2}{\beta} \log\left(I_0\left(\frac{2\beta\alpha}{\sigma^2}\right)\right)$  represents the degradation caused by the transformation of a flat fading channel into a multipath one.

Therefore, the ESG will be positive (in dB) if

$$e^{C\alpha^2} > I_0(2C\alpha) \quad (20)$$

with  $C = \frac{\beta}{\sigma^2}$ .

#### B. Rician Channel

In this case, both channels have a Line Of Sight (LOS) and a Non Line Of Sight (NLOS) component.

First, we will study the single transmitter case, with  $H_k \sim \mathcal{CN}(\mu_h, \sigma_h^2)$ . Under the sufficiently large number of carriers assumption, we have that

$$\Theta(\hat{\gamma}_S) = \frac{1}{N} \sum_{k=1}^N \Theta\left(\frac{|H_k|^2}{\sigma^2}\right) \approx E_x \left\{ \Theta\left(\frac{x^2}{\sigma^2}\right) \right\} \quad (21)$$

where  $x$  is Rician distributed with parameters  $\nu = |\mu_h|$  and  $\sigma_x^2 = \sigma_h^2/2$ . Therefore, we can write

$$\begin{aligned} \Theta(\hat{\gamma}_S) &= \int_0^\infty \sum_{l=1}^L \phi_l e^{-\frac{\beta_l x^2}{\sigma^2}} f_x(x) dx \\ &= \sum_{l=1}^L \phi_l e^{-\frac{\nu^2}{2\sigma_x^2}} \frac{1}{\sigma_x^2} \int_0^\infty x e^{-\frac{x^2}{\sigma_x^2} \left(\frac{2\beta_l}{\sigma^2} + \frac{1}{\sigma_x^2}\right)} I_0\left(\frac{x\nu}{\sigma_x^2}\right) dx. \end{aligned} \quad (22)$$

<sup>3</sup>Unfortunately, we are not able to provide closed form expressions for  $L > 1$  due to the impossibility of obtaining a closed form inverse for (7).

$$\begin{aligned}\Theta(\hat{\gamma}_H) &= \frac{1}{2\pi} \int_0^{2\pi} \sum_{l=1}^L \phi_l \frac{\sigma^2}{\beta_l \sigma_z^2 + \sigma^2} \exp\left(-\frac{\nu^2 \beta_l}{\beta_l \sigma_z^2 + \sigma^2}\right) da \\ &= \sum_{l=1}^L \phi_l \frac{\sigma^2}{\beta_l \sigma_z^2 + \sigma^2} \exp\left(-\frac{\beta_l(1+\alpha^2)}{\beta_l \sigma_z^2 + \sigma^2}\right) I_0\left(\frac{2\beta_l \alpha}{\beta_l \sigma_z^2 + \sigma^2}\right).\end{aligned}\quad (30)$$

$$\Delta\hat{\gamma} = \frac{\frac{1}{\beta} \log\left(1 + \beta \frac{1+\alpha^2}{K\sigma^2}\right) + \frac{1+\alpha^2}{\beta \frac{1+\alpha^2}{K} + \sigma^2} - \frac{1}{\beta} \log\left(I_0\left(\frac{2\beta\alpha}{\beta \frac{1+\alpha^2}{K} + \sigma^2}\right)\right)}{\frac{1}{\beta K^{-1} + \sigma^2} + \frac{1}{\beta} \log\left(1 + \beta \frac{K^{-1}}{\sigma^2}\right)} \quad (32)$$

Now, if we denote  $p_l = \sqrt{\frac{2\beta_l}{\sigma^2} + \frac{1}{\sigma_x^2}}$ ,  $a = \frac{\nu}{\sigma_x^2}$ , we can follow [9] and write

$$\int_0^\infty x e^{-p_l^2 x^2/2} I_0(ax) dx = \frac{1}{p_l^2} e^{\frac{a^2}{2p_l^2}} Q(a/p_l, 0) = \frac{1}{p_l^2} e^{\frac{a^2}{2p_l^2}} \quad (23)$$

with  $Q(x, 0) = 1 \forall x$  the Marcum Q-function, so (22) can be written as

$$\Theta(\hat{\gamma}_S) = \sum_{l=1}^L \phi_l \frac{1}{\sigma_x^2 p_l^2} \exp\left(-\frac{\nu^2}{2\sigma_x^2} \left(1 - \frac{1}{\sigma_x^2 p_l^2}\right)\right), \quad (24)$$

or, equivalently

$$\Theta(\hat{\gamma}_S) = \sum_{l=1}^L \phi_l \frac{\sigma^2}{\beta_l \sigma_h^2 + \sigma^2} \exp\left(-\frac{\nu^2 \beta_l}{\beta_l \sigma_h^2 + \sigma^2}\right). \quad (25)$$

Note that if we force  $\sigma_h^2 = 0$ , we arrive to the expression for the AWGN channel (12).

As in the AWGN case, we will obtain a closed form expression for the EESM, in order to gain insight on the problem. In this case

$$\hat{\gamma}_S = \frac{\nu^2}{\beta \sigma_h^2 + \sigma^2} + \frac{1}{\beta} \log\left(1 + \beta \frac{\sigma_h^2}{\sigma^2}\right). \quad (26)$$

Now, if we define  $\bar{\gamma}_N \doteq \frac{\sigma_h^2}{\sigma^2}$  the average SNR due to the multipath component, and  $\bar{\gamma}_L \doteq \frac{\nu^2}{\sigma^2}$  the average SNR caused by the direct component, we can write

$$\hat{\gamma}_S = \frac{\bar{\gamma}_L}{\beta \bar{\gamma}_N + 1} + \frac{1}{\beta} \log(1 + \beta \bar{\gamma}_N), \quad (27)$$

so we can find two different contributions to the ESM:

- The LOS component  $\frac{\bar{\gamma}_L}{\beta \bar{\gamma}_N + 1}$  is similar to the one in AWGN, but in this case the NLOS contribution acts as an additional noise source (it could be thought as a *self-interference* term).
- The NLOS component  $\frac{1}{\beta} \log(1 + \beta \bar{\gamma}_N)$ , which was not present in the AWGN channel, provides a ESM gain that increases logarithmically with the SNR.

Now, we proceed to calculate the ESM in a hybrid SFN scenario. In this case, we have that  $H_k = \mathcal{CN}(1, \sigma_h^2)$  and  $G_k = \mathcal{CN}\left(\alpha e^{-j(\theta + 2\pi n_0 \frac{k}{N})}, \sigma_g^2\right)$ . Like in the AWGN case,

we will assume that the phase term in  $G_k$  conforms an approximately uniform sampling of the interval  $[0, 2\pi)$ , so we have  $a \sim \mathcal{U}[0, 2\pi)$ , and, therefore, the distribution of  $Z_k = G_k + H_k$  conditioned on  $a$  is  $Z_k \sim \mathcal{CN}(1 + \alpha e^{-ja}, \sigma_g^2 + \sigma_h^2)$ . With this, we can approximate

$$\Theta(\hat{\gamma}_H) = \frac{1}{N} \sum_{k=1}^N \Theta\left(\frac{|Z_k|^2}{\sigma^2}\right) \approx E_Z \left\{ \frac{|Z|^2}{\sigma^2} \right\}. \quad (28)$$

We will solve the expectation by conditioning on  $a$

$$\Theta(\hat{\gamma}_H) = \int_0^{2\pi} E_{Z|a} \left\{ \frac{|Z|^2}{\sigma^2} \middle| a \right\} f_a(a) da. \quad (29)$$

Now, note that  $E_{Z|a} \left\{ \frac{|Z|^2}{\sigma^2} \middle| a \right\}$  is a particular case of (21), so a closed form expression follows (25) with  $\nu = |1 + \alpha e^{-ja}|$  and the variance of the NLOS component is  $\sigma_z^2 = \sigma_g^2 + \sigma_h^2$  instead of  $\sigma_h^2$ . Therefore  $\Theta(\hat{\gamma}_H)$  can be obtained by just averaging over  $a$  as shown in (30). Finally,  $\hat{\gamma}_H$  for  $L = 1$  can be written as

$$\hat{\gamma}_H = \frac{1}{\beta} \log\left(1 + \beta \frac{\sigma_z^2}{\sigma^2}\right) + \frac{1 + \alpha^2}{\beta \sigma_z^2 + \sigma^2} - \frac{1}{\beta} \log\left(I_0\left(\frac{2\beta\alpha}{\beta \sigma_z^2 + \sigma^2}\right)\right) \quad (31)$$

which has three different terms:

- $\frac{1}{\beta} \log\left(1 + \beta \frac{\sigma_z^2}{\sigma^2}\right)$  reflects the ESM gain due to the NLOS component.
- $\frac{1+\alpha^2}{\beta \sigma_z^2 + \sigma^2}$  includes the power gain  $(1 + \alpha^2)$  due to the insertion of LOS component coming from the terrestrial transmitter, and includes the channel degradation due to the presence of an NLOS component of power  $\sigma_z^2$ .
- $\frac{1}{\beta} \log\left(I_0\left(\frac{2\beta\alpha}{\beta \sigma_z^2 + \sigma^2}\right)\right)$  reflects the channel degradation due to the LOS component of the CGC.

Finally, if we assume the same power ratio between the LOS and NLOS components in both transmitters (the  $K$  Rician factor), so  $H_k \sim \mathcal{CN}(1, K^{-1})$  and  $G_k \sim \mathcal{CN}\left(\alpha e^{-j(\theta + 2\pi n_0 \frac{k}{N})}, \alpha^2 K^{-1}\right)$ , then the ESG can be written as in (32).

#### IV. ALAMOUTI PREPROCESSING

The use of Alamouti Space Time Codes (STC) in Multiple Input Single Output (MISO) processing [10] can be used to



overcome the channel degradation problem. With this kind of precoding, present in state of the art standards like DVB-T2 [11], the terrestrial transmitter does not convey the same message as the satellite, but a slightly modified constellation point.

After the processing performed at the receiver, which requires to estimate the channel from both transmitters separately, the resulting SNR at the  $k$ -th carrier would be

$$\gamma_k = \frac{|H_k|^2 + |G_k|^2}{\sigma^2}. \quad (33)$$

Note that the two channel contributions are added after the modulus squared operation, as opposed to (8). This preprocessing ensures that the SNR at each carrier is greater than or equal to that in absence of the CGC, so a positive ESG is always attained.

Now, we will derive the expression for the ESM (5) in the Rician hybrid scenario (for the only-satellite scenario, the ESM follows (26)). Combining (5) and (33) we have that

$$\hat{\gamma}_H = \Theta^{-1} \left( \frac{1}{N} \sum_{k=1}^N \Theta \left( \frac{|H_k|^2 + |G_k|^2}{\sigma^2} \right) \right) \quad (34)$$

so, if we assume a sufficiently large number of carriers, we can write

$$\Theta(\gamma_H) = E_{H,G} \left\{ \Theta \left( \frac{|H|^2 + |G|^2}{\sigma^2} \right) \right\} \quad (35)$$

with  $H \sim \mathcal{CN}(1, \sigma_h^2)$  and  $G \sim \mathcal{CN}(\alpha, \sigma_g^2)$ . Substituting  $\Theta$  by its approximation (7) we arrive to

$$\begin{aligned} \Theta(\gamma_H) &= \sum_{l=1}^L \phi_l E_{H,G} \left\{ e^{-\beta_l |H_k|^2} e^{-\beta_l |G_k|^2} \right\} \\ &= \sum_{l=1}^L \phi_l E_H \left\{ e^{-\beta_l |H_k|^2} \right\} E_G \left\{ e^{-\beta_l |G_k|^2} \right\} \end{aligned} \quad (36)$$

since  $H$  and  $G$  are assumed to be independent. The two expected values in (36) can be solved following the same procedure as in (21) to obtain

$$\begin{aligned} \Theta(\gamma_H) &= \sum_{l=1}^L \phi_l \frac{\sigma^2}{\beta_l \sigma_h^2 + \sigma^2} \exp \left( -\frac{\beta_l}{\beta_l \sigma_h^2 + \sigma^2} \right) \times \\ &\quad \times \frac{\sigma^2}{\beta_l \sigma_g^2 + \sigma^2} \exp \left( -\frac{\beta_l \alpha^2}{\beta_l \sigma_g^2 + \sigma^2} \right). \end{aligned} \quad (37)$$

If we set  $L = 1$  a closed form expression for  $\gamma_H$  can be found as

$$\begin{aligned} \gamma_H &= \frac{1}{\beta} \log \left( 1 + \beta \frac{\sigma_h^2}{\sigma^2} \right) + \frac{1}{\beta \sigma_h^2 + \sigma^2} + \\ &\quad + \frac{\alpha^2}{\beta \sigma_g^2 + \sigma^2} + \frac{1}{\beta} \log \left( 1 + \beta \frac{\sigma_g^2}{\sigma^2} \right) \\ &= \hat{\gamma}_S + \hat{\gamma}_T \end{aligned} \quad (38)$$

with

$$\hat{\gamma}_T \doteq \Theta^{-1} \left( \frac{1}{N} \sum_{k=1}^N \Theta \left( \frac{|G_k|^2}{\sigma^2} \right) \right) \quad (39)$$

the ESM of an only-CGC scenario. It is clear that, since

$$\Delta \hat{\gamma} = 1 + \frac{\hat{\gamma}_T}{\hat{\gamma}_S} \geq 1. \quad (40)$$

the system performance is always improved.

Although it can seem that this distributed MISO processing clearly solves the channel degradation problem, it presents some serious drawbacks:

- **Standard dependency** Receivers have to be designed to be able to perform the necessary processing in order to properly obtain the transmitted symbols, thus requiring that the standard they are based on includes Alamouti STC as an option. If this is not the case, the standard should be updated to include it, which is usually undesirable. For example, DVB-SH does not support this kind of MISO processing.
- **Increased overhead** As the receiver has to estimate the channels with both the terrestrial transmitter and the satellite, the pilot density has to be doubled with respect to the SISO operation, which significantly increases the signaling overhead.
- **Increased complexity** The receiver has to include additional hardware, mainly the presence of additional multipliers for the Alamouti processing, and the duplication of the channel estimation stage.
- **Extension to more transmitters** The extension of the Alamouti STC (which is a full-diversity rate-one STC) to more than two transmitters (or transmit antennas) is not possible without a rate loss [12]. Moreover, the pilot density should be increased proportionally to the number of transmitters, which is not scalable in practice.

In the next section we will introduce an alternative method to avoid the channel degradation problem that, despite not achieving the same gain as the Alamouti preprocessing, does not present the previous drawbacks.

## V. PRE-FILTERING

In this section we will explain how an appropriate filtering at the CGC can improve the performance of the system and avoid the channel degradation problem. We will illustrate the usefulness of this filtering for the AWGN channel, although simulations will be performed for general Rician channels.

If we assume an uniform phase distribution in (13), we can write

$$T_k \doteq |H_k + G_k|^2 = 1 + \alpha^2 + 2\alpha \cos(a_k) \quad (41)$$

with  $a_k \sim \mathcal{U}(0, 2\pi]$  independent and identically distributed. Note that the channel degradation problem is caused by some of the  $T_k$  suffering a *destructive interference* so  $T_k < 1 = |H_k|^2$ . In fact, as explained in Section III, the values of  $T_k$  are contained in the interval  $[(1 - \alpha)^2, (1 + \alpha)^2]$ , so  $T_k > 1 = |H_k|^2 \forall K$ , i.e., an SNR gain in every carrier can be assured if  $\alpha \geq 2$ . Unfortunately, the parameter  $\alpha$  cannot be modified by the CGC, as it would require to increase the transmit power or change the CGC location, which is usually not possible.

However, if  $\alpha < 2$ , the terrestrial transmitter could perform a filtering in the time domain (or, alternatively, a power

$$\hat{\gamma}_H = -\frac{1}{\beta} \log \left( (1 - \varphi) \frac{\sigma^2}{\beta\sigma_h^2 + \sigma^2} \exp \left( -\frac{\beta}{\beta\sigma_h^2 + \sigma^2} \right) + \varphi \frac{\sigma^2}{\beta\sigma_z^2 + \sigma^2} \exp \left( -\beta \frac{1 + \frac{\alpha^2}{\varphi}}{\beta\sigma_z^2 + \sigma^2} \right) I_0 \left( \frac{2\beta(\alpha/\sqrt{\varphi})}{\beta\sigma_z^2 + \sigma^2} \right) \right) \quad (50)$$

$$\Delta\hat{\gamma} = \frac{-\frac{1}{\beta} \log \left( (1 - \varphi) \frac{\sigma^2}{\beta K^{-1} + \sigma^2} \exp \left( -\frac{\beta}{\beta K^{-1} + \sigma^2} \right) + \varphi \frac{\sigma^2}{\beta K^{-1}(1 + \frac{\alpha^2}{\varphi}) + \sigma^2} \exp \left( -\beta \frac{1 + \frac{\alpha^2}{\varphi}}{\beta K^{-1}(1 + \frac{\alpha^2}{\varphi}) + \sigma^2} \right) I_0 \left( \frac{2\beta(\alpha/\sqrt{\varphi})}{\beta K^{-1}(1 + \frac{\alpha^2}{\varphi}) + \sigma^2} \right) \right)}{\frac{1}{\beta K^{-1} + \sigma^2} + \frac{1}{\beta} \log \left( 1 + \beta \frac{K^{-1}}{\sigma^2} \right)} \quad (51)$$

weighting in the DFT domain) so the power is concentrated in a fraction of carriers. If we denote by  $F_k$  the DFT response of the filter, the channel model (2) reads for the  $k$ -th carrier as

$$Y_k = (H_k + G_k F_k) X_k + W_k = (1 + e^{ja_k} \alpha F_k) X_k + W_k. \quad (42)$$

We will consider the following structure for the filter  $F_k$ :

- A fraction  $1 - \frac{\alpha^2}{4}$  of the carriers will be weighted with  $F_k = 0$ . As the ESM does not depend on the particular position of this carriers, we will assume, without loss of generality, that the first  $N(1 - \frac{4}{\alpha^2})$  carriers are nulled by the transmit filter  $F_k$ . Obviously, the power spent on these carriers is zero, and  $T_k = |H_k|^2 = 1$ .
- The remaining carriers (this is, a fraction  $\varphi \doteq \frac{\alpha^2}{4}$  of them) are weighted with  $F_k = \frac{2}{\alpha}$ . The average power consumption of this group of carriers is  $\frac{4}{\alpha^2}$ , and we have that

$$T_k = |1 + 2e^{ja_k}|^2 \geq 1 = |H_k|^2. \quad (43)$$

Note that with this approach we assure that no carrier suffers a power loss ( $T_k \geq 1 \forall k$ ), and the transmit power is not increased, as

$$\frac{1}{N} \sum_{k=1}^N |F_k|^2 = \varphi \frac{4}{\alpha^2} = 1, \quad (44)$$

so the power consumption is the same as in the absence of  $F_k$  (or, equivalently,  $F_k = 1 \forall k$ ).

This filter was shown in [13] to be optimal in the high SNR regime for the AWGN channel, while for lower SNR values the two-level filter was shown to be optimal, but the optimum fraction of active carriers is no longer  $\frac{\alpha^2}{4}$  and has to be computed numerically.

Now, we proceed to calculate the ESM of a hybrid system where the CGC uses this kind of filtering. Similarly to (8), we have

$$\begin{aligned} \hat{\gamma}_H &= \Theta^{-1} \left( \frac{1}{N} \sum_{k=1}^N \Theta \left( \frac{|H_k + F_k G_k|^2}{\sigma^2} \right) \right) \quad (45) \\ &= \Theta^{-1} \left( \frac{1}{N} \left( \sum_{k=1}^{N\varphi} \Theta \left( \frac{|1 + 2e^{ja_k}|^2}{\sigma^2} \right) + \sum_{k=N\varphi+1}^N \Theta \left( \frac{1}{\sigma^2} \right) \right) \right). \end{aligned}$$

Once again, if we assume a sufficiently large number of carriers<sup>4</sup>, we can approximate (45) by

$$\begin{aligned} \Theta(\hat{\gamma}_H) &\approx (1 - \varphi) \sum_{l=1}^L \phi_l e^{-\frac{\beta_l}{\sigma^2}} + \varphi \sum_{l=1}^L \phi_l E_a \left\{ e^{-\frac{\beta_l(1+2e^{ja})}{\sigma^2}} \right\} \quad (46) \\ &= \sum_{l=1}^L \phi_l \exp \left( -\frac{\beta_l}{\sigma^2} \right) \left( (1 - \varphi) + \varphi \exp \left( -\frac{4\beta_l}{\sigma^2} \right) I_0 \left( \frac{4\beta_l}{\sigma^2} \right) \right). \end{aligned}$$

Note that in the only-satellite scenario we have

$$\Theta(\hat{\gamma}_S) = \sum_{l=1}^L \phi_l \exp \left( -\frac{\beta_l}{\sigma^2} \right), \quad (47)$$

and since  $e^x \geq I_0(x)$  [14],

$$(1 - \varphi) + \varphi \exp \left( -\frac{4\beta_l}{\sigma^2} \right) I_0 \left( \frac{4\beta_l}{\sigma^2} \right) \leq 1, \quad (48)$$

we have that  $\hat{\gamma}_H \geq \hat{\gamma}_S$ , as  $\Theta$  is a monotonic decreasing function.

Like in the previous cases, we will provide a closed-form expression for  $L = 1$ . It can be easily seen that the EESM reads as

$$\hat{\gamma}_H = \frac{1}{\sigma^2} - \frac{1}{\beta} \log \left( 1 - \varphi + \varphi \exp \left( -\frac{4\beta_l}{\sigma^2} \right) I_0 \left( \frac{4\beta_l}{\sigma^2} \right) \right) \quad (49)$$

and the ESG is

$$\Delta\hat{\gamma} = 1 - \frac{\sigma^2}{\beta} \log \left( 1 - \varphi + \varphi \exp \left( -\frac{4\beta_l}{\sigma^2} \right) I_0 \left( \frac{4\beta_l}{\sigma^2} \right) \right) \quad (50)$$

which is always greater than or equal to one, since (48) holds.

This result for the AWGN channel can be extended to a general Rician channel following similar steps. The ESM of the hybrid system follows (50), while the ESG assuming the same Rician  $K$  factor from the terrestrial transmitter and the satellite reads as (51). These two formulas can be obtained following the same steps that led to (31) and (32).

This preprocessing offers some clear advantages with respect to the Alamouti preprocessing, but it also presents some drawbacks

- The power gain is smaller than the one attained with Alamouti STC.

<sup>4</sup>In this case, the *sufficiently large* approximation must hold not only for the overall system, but also for each of the two groups of carriers.

- The fraction of active carriers  $\varphi$  depends on the value of  $\alpha$  (the relative amplitude between the CGC and satellite components) and, therefore, on the position of the receiver. The design of the filter is more involved if several receivers with a huge range of values of  $\alpha$  are present.

## VI. RESULTS

In this section we will evaluate the derived analytical expressions and verify them by means of simulations. All the simulations were conducted for  $L = 1$ ,  $\beta = \beta_{QPSK} \approx 0.65$  the parameter resulting from the Minimum Mean Square Error (MMSE) fitting of (6) by (7) for a QSPK constellation, and  $N = 1024$  carriers. Although the derivations were performed assuming independence between carriers, in the simulations we will generate the channel in the time domain with a limited length, as the result of assuming an overall channel shorter than the CP length. The simulations were performed with a CP length of  $N/4$ .

First of all, we will illustrate the effect of the different approaches in the resulting channel seen by a given receiver. In Figure 2 there is a plot of the channels obtained with the different approaches in a scenario with  $\alpha = 1$ , i.e., same power coming from the satellite and the CGC, and  $K = 25dB$ , which can be the case of a system operating in the S-band in an open environment with LOS reception [15, Table VII]. As we are assuming an almost pure LOS scenario, the squared modulus of the satellite and CGC channels is similar for all carriers, and approximately equal to 1 (these channels are not shown for the sake of clarity). The effects on the channel of the different preprocessing strategies are:

- With no preprocessing, the channels are directly added *in the air* with different phases, thus resulting in a sequence of carriers suffering negative and positive interference. As both satellite and CGC channels are approximately equal to 1, the squared modulus of the sum channel is concentrated between  $\max_k \{|H_k + G_k|^2\} \approx 4$  and  $\min_k \{|H_k + G_k|^2\} \approx 0$ . It is clear that those carriers with  $|H_k + G_k|^2 < 1$  will achieve a lower SNR than in the only-satellite scenario, thus potentially harming the performance of the system.
- With the proposed pre-filtering, part of the carriers are nulled at the CGC, so the resulting channel is equal to that in the only-satellite scenario. All the available power is concentrated in a group of carriers (from  $k = 769$  to  $k = 1024$ ), being the filter coefficient in these carriers set to  $F_k = 2$ . Similarly to the non-preprocessing case, the channels are added prior to the squared-modulus operation, so the resulting channel is concentrated between  $\max_k \{|H_k + F_k G_k|^2\} \approx 9$  and  $\min_k \{|H_k + F_k G_k|^2\} \approx 1$ . Note that in this case the minimum value is approximately equal to 1, which was the channel value in the only-satellite scenario, so no carrier suffers an SNR loss (up to the random but weak multipath component).
- With Alamouti preprocessing, the squared modulus of both channels are added following (33). Therefore, the

resulting channel is almost flat (except for the weak multipath component), and approximately equal to 2, as the result of  $|H_k|^2 + |G_k|^2 \approx 2$ .

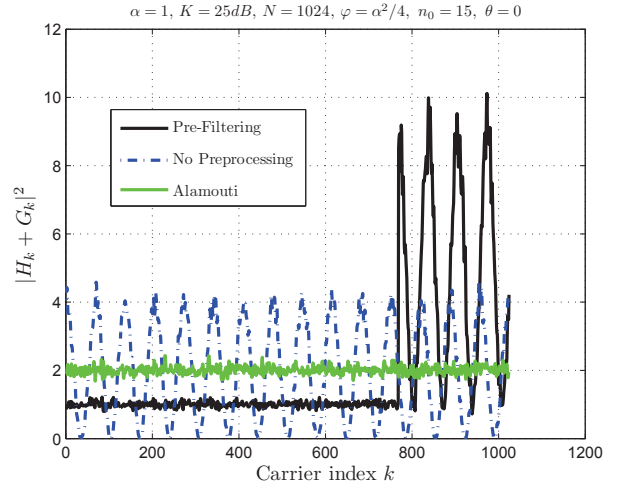


Fig. 2. Hybrid channel response without preprocessing, with pre-filtering and with Alamouti coding.

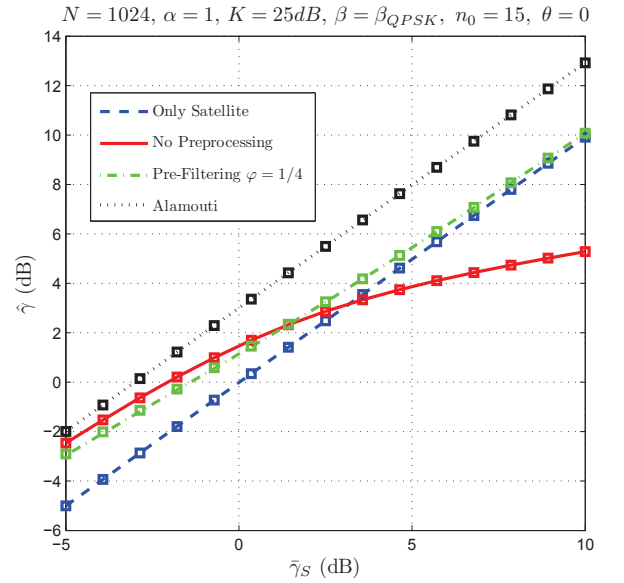


Fig. 3. Analytical (squares) and simulated (lines) effective SNR as a function of the initial average SNR in the presence of one and two transmitters for different scenarios.

In Figure 3 there is a plot of the analytical and simulated ESM as a function of the average SNR in absence of the CGC for the different approaches. It can be seen that the Alamouti preprocessing clearly outperforms the other strategies, specially for high SNR, and that the insertion of the CGC with no preprocessing leads to a lower ESM than the only-satellite scenario (this implies that the channel degradation is larger than the power gain). For low SNR values the channel degradation is not that important, and the insertion

of a terrestrial transmitter clearly increases the performance of the system.

The Pre-Filtering approach provides a smaller gain than the Alamouti preprocessing, but can be seen to clearly avoid the channel degradation problem, as the Pre-Filtering curve is always above the No Preprocessing one. It is also remarkable that the analytical approximations provide an almost perfect prediction of the ESM.

In Figure 4 we present similar results for a scenario with a stronger multipath component ( $K = 5dB$ , that could arise in a heavy tree shadow environment with intermediate shadowing [15, Table VII]). In this case we are representing the ESG  $\Delta\hat{\gamma}$  instead of the ESM  $\hat{\gamma}$  for different values of fraction of active carriers  $\varphi$  as a function of the average SNR in an only-satellite scenario. It can be seen that the selection of  $\varphi$  is quite involved, as it depends on the SNR working point: for low SNR  $\varphi = 1$  provides a higher gain, for medium SNR (around  $5dB$ ) the curve  $\varphi = 3/4$  is above the others, while for high SNR the best option is  $\varphi = 1/4$ . It is also noticeable that the losses due to the channel degradation are much smaller than the ones shown in Figure 3: note that for  $\bar{\gamma}_S = 10dB$  there is a loss of approximately  $0.25dB$  for the No Preprocessing curve, while Figure 3 shows a loss of approximately  $5dB$  for the same SNR value.

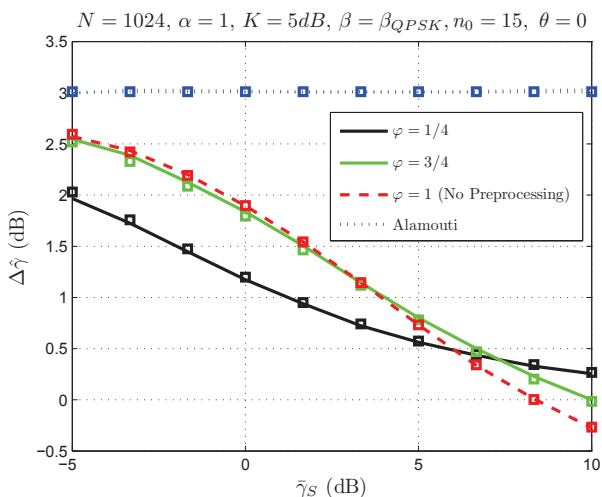


Fig. 4. Analytical (squares) and simulated (lines) effective SNR as a function of the initial average SNR for different scenarios. The Pre-filtering is performed for different values of  $\varphi$ .

## VII. CONCLUSIONS

In this paper we have derived analytical expressions for the performance characterization of a hybrid terrestrial-satellite single frequency network using effective SNR metrics. Focusing on those cases where the power contributions from the terrestrial transmitter and the satellite are similar, we conclude that the *channel degradation* due to the presence of echoes has to be taken into account when designing these kind of networks, specially in high SNR and strong LOS scenarios.

Additionally, we have presented two different approaches to overcome this performance loss, which are also characterized by closed form formulas, and detailed the main system-level implications for each of them.

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